PROJECTION THEOREMS IN RIEMANNIAN MANIFOLDS

ANNINA ISELI

ABSTRACT. A theorem of Marstrand states that for a given compact set A in the Euclidean plane of Hausdorff dimension smaller or equal to 1, the set we receive by projecting A orthogonally onto a generic line through the origin has the same Hausdorff dimension as A. We will recall some similar but more general results in Euclidean space and briefly review different methods for proving them. It turns out that the statements of these theorems can be formulated also for many (negatively curved) Riemannian manifolds. We will discuss which methods might apply to different Riemannian settings and what sort of difficulties arise.

UNIVERSITY OF BERN

HAUSDORFF DIMENSION AND EXCEPTIONAL PROJECTIONS AND INTERSECTIONS

PERTTI MATTILA

Abstract. According to Marstrand's theorem from 1954 a Borel subset of the plane of Hausdorff dimension s at most 1 projects into a set of dimension s in almost all directions. Kaufman gave a sharper version in 1968 saying that the set of exceptional directions is at most s. There are other similar results and more recent exceptional set results, by Orponen and myself, for planar sections and general intersections. I shall discuss some of these.

University of Helsinki

UNIFORMIZATION OF SIERPINSKI CARPETS BY SQUARE CARPETS

DIMITRIOS NTALAMPEKOS

Abstract. Uniformization of metric spaces is the problem of finding conditions on a metric space, under which it can be transformed to a canonical space, by a map that preserves the geometry. In this talk, our metric space will be planar Sierpinski carpets, and the canonical spaces are square Sierpinksi carpets. We prove that every Sierpinski carpet, under certain geometric assumptions, can be mapped by a quasisymmetric map to a square carpet. This is achieved with the aid of carpet-harmonic functions, which is a discrete notion of harmonic functions on Sierpinski carpets.

University of California UCLA

RIGIDITY FOR THE SPECTRAL GAP ON RCD(K, ∞)-SPACES

SHIN-ICHI OHTA

Abstract. We consider a rigidity problem for the spectral gap of Laplacian on an RCD(K, ∞)-space (a metric measure space satisfying the curvature-dimension condition) for positive K. For a weighted Riemannian manifold, Cheng-Zhou (2013) showed that the sharp spectral gap is achieved only when a 1-dimensional Gaussian space is split off. Generalizing this to RCD-spaces is not straightforward due to the lack of smooth structure as well as the lack of upper dimension bound. In order to overcome these difficulties, we lift an eigenfunction to the Wasserstein space and employ the regular Lagrangian flow recently developed by Ambrosio-Trevisan. This is a joint work with N. Gigli (SISSA), C. Ketterer (Freiburg) and K. Kuwada (Tohoku).

Kyoto University

BESICOVITCH COVERING PROPERTY ON GRADED GROUPS AND APPLICATIONS TO MEASURE DIFFERENTIATION

SÉVERINE RIGOT

Abstract. In this talk we will give a characterization of graded groups admitting homogeneous distances for which the Besicovitch covering property (BCP) holds. In particular, a stratified group admits homogeneous distances for which BCP holds if and only if the group has step 1 or 2. On the other hand, we will stress that on a stratified group of step 2, such homogeneous distances for which BCP holds are never given by sub-Riemannian distances. We will also discuss some applications to measure differentiation which is one of the motivations for considering such covering properties.

University Nice Sophia Antipolis

SUB-RIEMANNIAN GEOMETRIC INEQUALITIES

LUCA RIZZI

Abstract. We discuss how to obtain interpolation inequalities for general ideal sub-Riemannian structures (i.e. with no non-trivial abnormal minimizers). As a byproduct, we characterize the sub-Riemannian cut locus as the set of points where the squared sub-Riemannian distance is not semiconvex. In particular, we deduce that the singularities of the distance for generic sub-Riemannian structures are the same occurring in Riemannian case, answering - in the generic case - to a question raised by Figalli and Rifford. Specifying our results to the case of the Heisenberg groups, we recover the inequalities recently obtained by Balogh, Kristly and Sipos. The techniques are based on optimal transport and sub-Riemannian Jacobi fields. This is a joint work with D. Barilari.

CNRS - Institut Fourier (Grenoble)

JACOBIAN DETERMINANT INEQUALITY ON HEISENBERG GROUPS

KINGA SIPOS

Abstract. Contrary to former belief, we established on the Heisenberg group $\mathbb{H}^n$ a determinant inequality holding for the Jacobians of the optimal mass transportation map and s-intermediate optimal mass transportation map, both corresponding to the quadratic cost function. Our proof is based on the Riemannian approximation of $\mathbb{H}^n$ and on the corresponding Jacobian determinant inequality on the Riemannian manifolds (due to Cordero-Erausquin, McCann and Schmuckenschläger). Our Jacobian determinant inequality proves to be an equivalent formulation of the famous curvature dimension condition due to Lott, Villani and Sturm. Furthermore, we showed that this determinant inequality implies a set of geometric inequalities, like measure contraction property, a geodesic version of the Borell-Brascamp-Lieb inequality (which in a particular case provides exactly the Prékopa-Leindler inequality) and Brunn-Minkowski inequalities.

University of Bern

ISOMETRIC EMBEDDINGS OF HEISENBERG GROUPS

HERNANDO SOBRINO

ABSTRACT. An important issue in the field of functional analysis raised by the polish mathematician Stefan Banach during the first half of the 20th century was the connection between isometric embeddings and the linearity property: under which circumstances are isometric embeddings of real normed vector spaces affine linear maps? The result obtained was that any isometric embedding is affine, as long as it is surjective or the target space is strict convex. In recent years, similar results have been obtained for isometric embeddings of Heisenberg Groups: any isometric embedding of two Heisenberg Groups equipped with homogeneous distances is the composition of a left translation and a homogeneous homomorphism, as long as it is surjective or the target space is "geodesic linear".

UNIVERSITY OF BERN

KINETIC ENERGY AS SUM OF MOMENTS OF MEASURE CORRESPONDING TO VORTICITY

MARTA SZUMANSKA

Abstract. We investigate properties of measures in the plane satisfying the following condition: for some $C > 0$ and $d > 0$ measure of every ball centered at zero with radius r is equal Cr^d.

Motivation of our studies are irregular flows in a plane whose vorticity is a measure supported on a curve. Our start point are examples suggested by Prandtl and Kaden (in '30 of the previous century), where in each time $t > 0$ vorticity of the flow satisfies the abovementioned condition.

We give a formula for kinetic energy of such flows which allows us to show that velocity field generated by Kaden spiral is well defined almost everywhere, its kinetic energy is locally finite, and kinetic energy of balls centered at zero is a decreasing function.

The talk is based on joint work with Tomasz Cieślak, Krzysztof Oleszkiewicz and Marcin Preisner

IMPAN, Warsaw

CALIBRATIONS AND CALIBRATIONS MODULO 2

ROGER ZÜST

Abstract. To show that a given oriented submanifold has minimal volume among all submanifolds with a fixed boundary or homology class is hard in general. The tool most often used for testing such minimality are calibrations. These are closed differential forms with pointwise unit norm. The 1-dimensional problem asks for the length minimal filling of a given, say finite, set of sources and drains. The Kantorovich duality of optimal mass transport guarantees the existence of calibrations in this situation. Similar to the oriented minimal filling problem one may be interested in the unoriented one, where coefficients modulo 2 are considered. In dimension one this asks for a minimal matching among an even number of points. In a joint work with Mircea Petrache we obtained a version of calibrations for this case.

The goal of this talk is to introduce classical calibrations and give some examples thereof before we discuss calibrations for the oriented and unoriented 1-dimensional minimal filling problem.

University of Bern